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Some (Known) Facts on Multicarrier Communication

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Abstract

It is widely known and accepted that multicarrier communication is effective in frequency selective channels if resource allocation strategies are used, namely per-subcarrier adaptive modulation and coding. We discuss its efficiency for uniform resource allocation in terms of fundamental information-theoretic measures. These results are relevant to broadcast systems and, more generally, systems which cannot rely on a feedback channel.

Index Terms

Multicarrier Communication, Orthogonal Frequency Division Multiplexing (OFDM), Information Rate, Broadcast Systems.

I. Introduction

MULTICARRIER (MC) communication, typically based on orthogonal frequency division multiplexing (OFDM), has established itself as an efficient and convenient method in most scenarios, including wireline, powerline and wireless systems. MC systems are currently being intensively investigated and experimented in optical communication as well. The efficiency of the MC approach in terms of both performance and complexity is out of discussion in several communication scenarios, so the common belief tends to view it as a panacea. However, a closer look at some fundamental features of the MC approach reveals that it may exhibit inefficiencies in specific scenarios.

In this respect, a recently investigated communication scenario is a channel impaired by impulse noise, which can be considered as a limiting factor in many applications, such as, for example, powerline, digital subscriber line (DSL), and wireless communication systems. Impulse noise typically originates from electrical and electromagnetic equipments and affects the transmission in the form of random bursts of relatively short duration and very high instantaneous power. Among a number of aspects, an element which played a role in the establishment of MC modulations was their improved robustness in impulse noise-limited communications, with respect to single-carrier (SC) schemes [1]. However, it has recently been shown that standard MC systems, employing interleaving and channel coding, are indeed less robust to impulse noise than corresponding SC schemes [2]. Interestingly, this result does not contradict previous findings obtained for uncoded systems, which show that MC schemes may outperform SC ones [1], because the loss manifests itself at rates of typical coded systems.

The results in [2] are obtained analyzing the ultimate performance limits of SC and MC communication systems in terms of the achievable information rate (IR). From an engineering viewpoint, this performance measure is appealing because it provides a realistic information-theoretic benchmark in the fact that it takes inherently into account specific design constraints, such as the modulation format or the transmitted power spectrum. Following this approach to performance evaluation in terms of fundamental measures, this paper uses the achievable IR to analyze and compare SC and MC schemes in frequency selective additive white gaussian noise (AWGN) channels.

II. System Model and Performance Measure

We consider the SC and MC system models shown in Fig. 1, parts (a) and (b), respectively. The information sequence $\mathcal{A}$ is input to quadrature amplitude modulation (QAM) devices. In the SC system, the sequence of QAM independent symbols $\mathcal{X}_{SC}$ is directly sent over a frequency selective (FS) channel, which adds a sequence $\mathcal{W}$ of independent identically distributed thermal noise samples. In the MC system, the QAM sequence is input to an inverse discrete Fourier transform (IDFT) device, whose output sequence $\mathcal{X}_{MC}$ is sent over the FS channel. The received sequences are denoted as $\mathcal{Y}_{SC}$ and $\mathcal{Y}_{MC}$.

For a general channel with discrete input $\mathcal{X}$ and continuous output $\mathcal{Y}$, the IR is defined as [3]

$$I(\mathcal{X};\mathcal{Y}) = h(\mathcal{Y}) - h(\mathcal{Y}|\mathcal{X}) \tag{1}$$

where $h(\mathcal{Y})$ denotes the differential entropy rate of the channel output process $\mathcal{Y}$, and $h(\mathcal{Y}|\mathcal{X})$ denotes the differential entropy rate of the channel output given the channel input. In particular, $h(\mathcal{Y}|\mathcal{X})$ equals the differential entropy rate of the noise

[(*)]Work performed while M. Franceschini and R. Pighi were at the University of Parma.

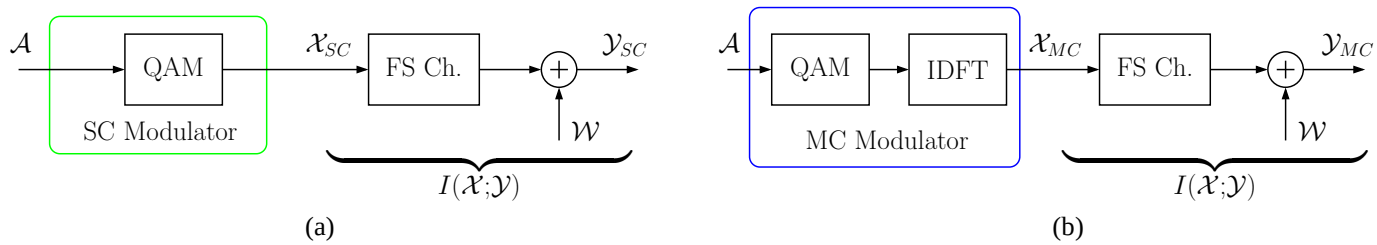

Fig. 1. SC (a) and MC (b) communication system models.

process $\mathcal{W}$, namely, $h(\mathcal{W}) = \log 2\pi e\sigma^2$, where σ^2 denotes the variance of a noise sample per real dimension. The IR of the noisy FS channel in Fig. 1 depends on the statistics of the input sequence $\mathcal{X}$, which differs in the SC and MC systems. We are then interested in analyzing and comparing the achievable IR for both systems. The reader is referred to [4] for details on the IR computation in SC and MC systems.

III. Results and discussion

We consider classic three-tap channels with normalized impulse responses $(1, 2, 1)/\sqrt{6}$ (channel *a*) and $(3, 2, 1)/\sqrt{14}$ (channel *b*). The frequency response of channel *a* is characterized by a second order zero at the normalized radian frequency π. Channel *b* is also FS, but exhibits a more "well-behaved" response with no in-band spectral zeros. Despite these channel models may appear artificial, they represent well the physical channels encountered in realistic broadband applications, characterized by strongly frequency selective attenuation, including wireline, powerline and wireless communications.

Fig. 2 shows the IR of SC and MC systems for channels *a* and *b* as a function of the signal-to-noise ratio (SNR) E_s/N_0, where E_s and N_0 denote the average energy per information symbol and the one-sided noise power spectral density, respectively. As a reference, the IR for a frequency flat AWGN channel is also shown (obviously, identical for SC and MC systems). The rate and energy losses incurred by MC systems due to the use of a cyclic prefix are not accounted for in Fig. 2. Hence, these results represent tight upper bounds on the IR for large numbers of subcarriers. If desired, the losses induced by a cyclic prefix could be easily accounted for, e.g., to avoid approximations for moderate numbers of subcarriers.

In Fig. 2, solid curves show the IR for a 16-QAM SC system, a MC system with uniform resource allocation employing 16-QAM in all subcarriers (curve labeled "no BL", namely relative to the absence of "bit loading"), and a MC system with optimized resource allocation transmitting an "average 16-QAM" over the subcarriers (curve labeled "WF + BL", namely relative to the use of "water filling" power allocation [3] and BL). The considered resource allocation algorithm assigns a WF power spectrum to the subcarriers and determines the BL scheme by maximizing the total IR for a given number of bits per OFDM symbol (which corresponds to an average 16-QAM) [5]. The dashed curves in Fig. 2 are descriptive of the IR for various systems which use high-order QAM with low code rates and possible signal shaping to approach a gaussian input (GI) distribution to the channel. Specifically, the ideal limits of an infinite QAM, namely a square QAM constellation with infinitely dense points, (denoted as "Inf. QAM") and a GI to the channel are considered. WF and uniform power (UP) spectrum at the input of the channel are also considered. Note that if a uniform power spectrum is transmitted, the unit energy normalization of the channel response allows us to equivalently interpret E_s as both transmitted and received symbol energy. However, for a non uniform transmitted spectrum this equivalence does not hold and E_s denotes the transmitted symbol energy, in agreement with the general results on water filling which define the transmitted power spectrum that maximizes the IR of a FS AWGN channel under a transmit power constraint [3].

Considering channel *a* and a reference IR of 3 bits/channel use, corresponding to a typical $3/4$ code rate and 16-QAM, Fig. 2(a) shows a 6 dB SNR penalty of the MC system with uniform resource allocation, as compared to the SC system (solid curves). Furthermore, this penalty may increase for higher code rates. However, a slight 0.4 dB SNR gain is obtained by the MC system with resource allocation, transmitting an average 16-QAM over the subcarriers, with respect to the 16-QAM SC system. The dashed curves shown in Fig. 2(a) demonstrate that the use of high-order constellations with low code rates and possible shaping also allows to avoid the SNR loss of MC systems and is an alternative to resource allocation (e.g., see the "Inf. QAM + UP" case). Nonetheless, for any finite QAM constellation (solid curves), a SC system may significantly outperform a MC system with uniform resource allocation if efficient code rates are of interest.

Fig. 2(b) shows the results of a similar analysis carried on for channel *b*. This channel has a less harsh frequency response in comparison with channel *a*, with no in-band spectral nulls. Hence, the difference in the performance of the various systems is less evident. Nonetheless, the conclusions are similar to those based on channel *a*. Considering a reference IR of 3.5 bits/channel use, corresponding to a typical $7/8$ code rate in a 4-dimensional trellis-coded modulation (TCM) scheme employing 16-QAM, Fig. 2(b) shows a 2.3 dB SNR penalty of the MC system with uniform resource allocation, as compared to the SC system. An appreciable 1.6 dB SNR gain is obtained by the MC system with resource allocation, transmitting an average 16-QAM over the subcarriers, with respect to the 16-QAM SC system. This gain is more significant than the slight one obtained for

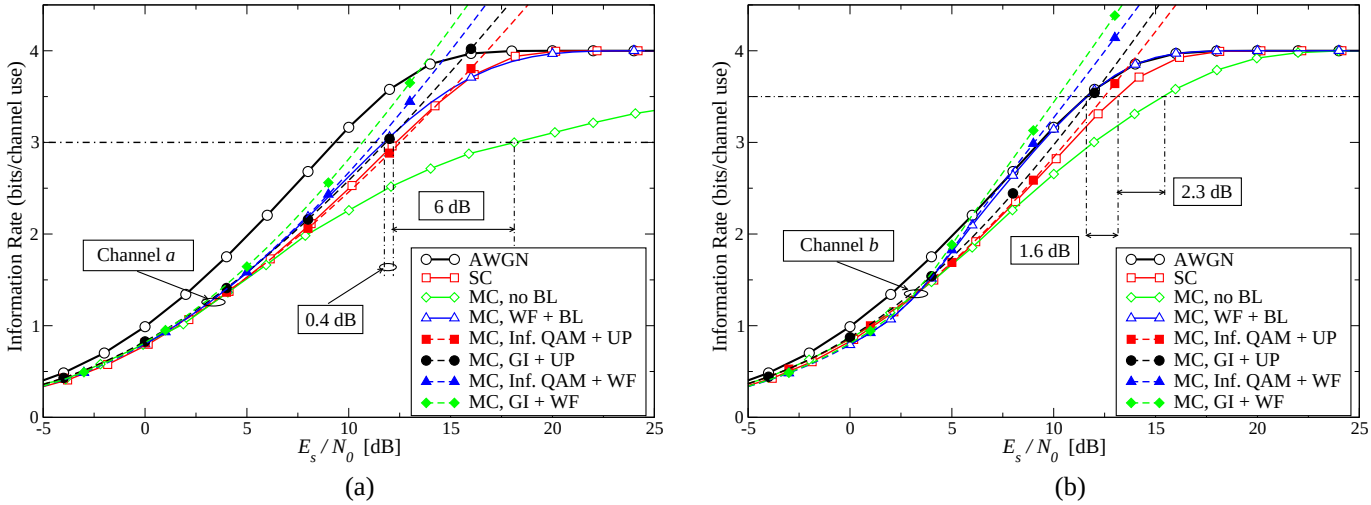

Fig. 2. IR versus SNR for various systems - (a): $(1, 2, 1)/\sqrt{6}$ channel; (b): $(3, 2, 1)/\sqrt{14}$ channel.

channel *a*. Results similar to those in Fig. 2(a) are also obtained for systems employing high-order QAM with low code rates and possible signal shaping.

The conclusions which can be drawn from the considered two illustrative channel models represent well the general case. Specifically, for any given constellation order, the loss of a MC system with uniform resource allocation with respect to a SC system is large for channels exhibiting deep spectral nulls, whereas it is less evident for channels with well-behaved frequency responses. An investigation reported in [4] demonstrates that this conclusion holds for any item of a number of randomly generated channels.

IV. Conclusion

SC may appreciably outperform MC systems with uniform resource allocation, for any fixed constellation order. This finding may be viewed as an information-theoretic counterpart of the known fact that WF and BL schemes are required in MC systems. It also raises some questions on the efficiency of MC schemes in broadcast applications, such as those defined by the terrestrial digital audio and video broadcasting standards (DAB and DVB-T), and more general applications which do not exploit resource allocation, such as those not relying on a feedback channel.

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